Minimum Weight Design of Aircraft Landing-Gear Reinforcement Rings

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In this paper, a method of designing cylinder-reinforcing rings for minimum weight is presented. The rings are idealized as planar frames with elastic supports at the joints. In each iteration step, the cross-section properties at the joints for the applied loads are defined. The forces and displacements at each member end are calculated by the stiffness method. The stresses and margins of safety at the ends of the members are calculated and the safety constraints formulated. The geometrical constraints and the weight of the rings are also established. A step is taken in the direction of feasible steepest descent to a new point at which the value of the objective function is improved. The iteration process is repeated until an optimum or a practical minimum is obtained. The procedure is illustrated by example.

Nomenclature

A_i = average cross-sectional area of the ith member, in.^{2}
F = the material weight, lb
$F(X)$ = objective function
G = modulus of shear rigidity of cylinder, psi
G_j = jth constraint
h = outer width of the section at the joint, in.
K = cylinder resisting force per unit tangential deflection, lb/in.
L = axial distance along the cylinder to a section which is not distorted from a circle (L is taken equal to R), in.
L_i = length of ith member, in.
M = margin-of-safety
m = number of variables
n = number of constraints
n_m = number of members
n_s = twice the number of members
p_i = ith applied load, lb
R = radius to center of cylinder wall, in.
R_a = axial stress ratio, i.e., the ratio of the applied axial stress to the ultimate tensile stress or to the compressive yield stress
R_b = bending stress ratio, i.e., the ratio of the applied bending stress to the ultimate bending stress; the ultimate bending stress is obtained by dividing the ultimate bending moment by the elastic section modulus
R_s = shear stress ratio, i.e., the ratio of the applied shear stress to the ultimate shear stress
t = average thickness of cylinder wall, in.
U = factor of utilization
w, y, z = parameters defining the thicknesses of the section at the joint, in.
X = design vector
x_i = ith design variable
$X^{(i)}$ = design vector in the ith iteration
λ = step length in the iteration process
ρ_i = density of the material of the ith member lb/in.^{3}

I. Introduction

Many aircraft landing-gear components consist of cylinders reinforced with rings at various locations. Torque collars and steering collars are typical examples of such reinforcements which for the purpose of this paper will be called cylinder-reinforcement rings.

It has been the accepted practice to design reinforcement rings by the "cut-and-try" method. In the cut-and-try method, a trial structure is analyzed and the stresses calculated at points along the ring. At points of low stresses, the corresponding cross sections can be reduced to eliminate excess weight. Conversely, larger cross sections can be assigned to points of high stress value, if necessary, and the structure then reanalyzed.

This paper presents a direct method of obtaining a least-weight design for reinforcement rings. The method leads to considerable weight savings, and the reduced solution time represents an improvement over the cut-and-try method. The described design procedure consists of the following steps. 1) The ring is idealized into a planar frame with elastic supports. 2) Given a set of cross-section properties and corresponding loads, the frame is analyzed and the forces at the ends of each member are determined. 3) The stresses and the margin-of-safety at each joint are calculated, thus establishing the safety constraints. 4) The weight of the ring material is expressed in terms of the design variables. 5) An optimization iteration step is performed and new values of the design variables are obtained. If the new design is optimum the solution process is completed. If an optimum design is not obtained, the following step is performed. 6) The cross-section properties are defined in terms of the new values of the variables. If the magnitude of the applied loads varies with the dimensions of the cross-sections, a new set of loads is calculated. 7) Steps 2–6 are repeated until an optimum design is obtained in step 5.

II. Idealization of the Ring

To analyze the ring, points thereon are selected and designated as joints. The joints are connected by straight members and the resulting polygon analyzed as a planar frame. The cross-sectional properties of each member are by definition identical to those of the corresponding arc on the real ring.

The effect of the supporting cylinder is considered as a tangential restraint.[1] This restraint is taken as

$$K = RtG/L \tag{1}$$

The restraint K corresponds to an arc length of one radian angle of the cylinder. The stiffness is assumed to be concentrated at each ring joint by considering half the arc length

Presented as Paper 68-328 at the AIAA/ASME 9th Structures, Structural Dynamics, and Materials Conference, Palm Springs, Calif., April 1–3, 1968; submitted March 25, 1968; revision received October 11, 1968.

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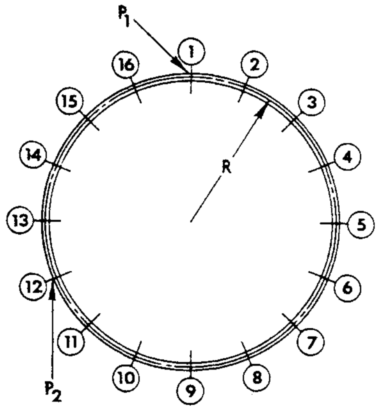

Fig. 1a Cylinder reinforcing ring.

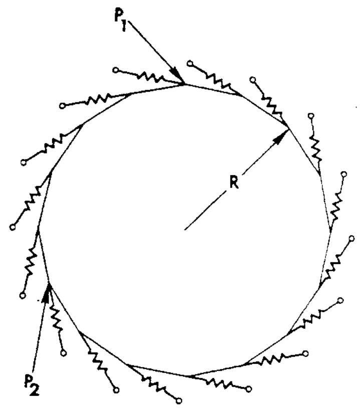

Fig. 1b Idealized ring.

on both sides of the joint. A tangential elastic spring with this stiffness is assumed to be attached to the ring at the given joint. The ring and its idealization are shown in Figs. 1a and 1b. The applied loads are represented by P_1 and P_2.

III. Frame Analysis

Given the applied loads and the cross-section properties for each member of the frame, the joint displacements are found by using the stiffness method. The stiffness matrices of the individual members are calculated and assembled into a frame-stiffness matrix. The latter matrix is inverted and the displacements are obtained. The forces at the end of each member are calculated using the displacements of the end joints and the member-stiffness matrix.

IV. Margin of Safety

The margin of safety at a given joint is a function of the combined stresses at the joint. In each iteration of the optimization process, the cross-sectional properties and the loads are determined for each joint. The factor of utilization is defined as

$$U = [(R_b + R_a)^2 + R_s{}^2]^{1/2} \tag{2}$$

Using a factor of safety equal to 1.5, the margin of safety is

$$M = (1/1.5U) - 1 \tag{3}$$

The strength constraint at the joint expresses the nonnegativity requirement on the margin of safety. This constraint is an implicit function of the design variables which define the cross section at the given joint.

Since the strength constraints are based on elastic analysis of the ring, the change in the values of the design variables leads to a change in the magnitude of the member forces. This change is neglected within each iteration, and the stresses are based on the member forces calculated in the previous iteration. This avoids reformulating the constraints within the iteration and reduces the solution time considerably.

V. Optimization

The design vector is defined as

$$X = (x_1, x_2, \ldots x_i, \ldots x_m) \tag{4}$$

This vector is established by assigning the required number of variables to each joint cross section. Symmetrical sections are assigned the same set of variables and fixed sections are not assigned any variables.

The strength requirement constraints are expressed in the following form:

$$G_j(X) \geq 0 \quad (j = 1, n_s) \tag{5}$$

Due to the elastic springs at the joints, the forces on the two sides of a joint are not equal. Therefore, the margin of safety is calculated at member ends rather than at joints.

The geometrical constraints express further design limitations. These constraints include upper and lower limits on the values of design variables related to a cross section in terms of other dimensions. These constraints are expressed as

$$G_j(X) \geq 0 \quad (j = n_s, n) \tag{5a}$$

The objective function to be minimized is the weight of the ring material, given by

$$F = \sum_{i=1}^{n_m} \rho_i A_i L_i \tag{6}$$

Each cross-sectional area is a function of the design variables. Thus, the objective function is also a function of the design variables.

Mathematically, the preceding problem is one of finding the set of variables $x_i (i = 1, m)$ which minimizes the function $F(X)$ subject to the set of constraints $G_j(X) \geq 0 (j = 1, n)$. An initial solution to the problem is required before commencing the optimization process. The initial solution can be a best-judgment estimate and need not necessarily be a feasible solution. Starting with such an initial solution, steps are then taken toward reduced values of the objective function until a minimum value is reached.

The direction of movement from an unconstrained feasible point in the direction of steepest descent is defined by the negative of the unit gradient of the objective function. At a constrained point, the admissible direction of steepest descent is obtained by sweeping out of the direction vector the components along the gradients of the active constraints. The step length is set as the distance to the first constraint encountered upon movement in the specified direction and is found by the method of binary chopping. This optimization concept was originally developed to permit design of frames for minimum cost of material and connections.[2]

As noted previously, infeasible initial solutions can also be used for the solution of the problem, thus eliminating the necessity of first obtaining feasible solutions. When constraints are violated at the occupied point, the resultant of the unit gradients of the violated constraints defines the direction of movement back to the feasible domain.

When a minimum value for the objective function is reached, the admissible direction of steepest descent is a zero vector and the iteration process ends. Since the rate of change of the objective function becomes less after a few successive iterations, a practical minimum can often be obtained by performing only a few iterations at a time and studying the resultant minimum values at the end of each succeeding series of iterations. If necessary, additional iterations can be performed, starting from the last solution, with minimum expenditure of computational time. If the solution obtained at the end of a number of iterations is considered by the designer to be close to optimum, the

Table 1 Initial values of the variables

Joint	y dimension, in.	z dimension, in.
1	0.60	0.40
2	0.60	0.40
3	0.60	0.40
4	0.60	0.40
5	0.60	0.40
6	0.60	0.40
7	1.00	0.60
8	2.00	0.85
9	2.25	1.00
10	2.15	1.00

computations may be terminated and that solution adopted as the final result. The change per iteration in the value of the objective function and the number of equality-satisfied constraints can serve as guides to aid the designer in determining when to terminate the computations. The use of practical minimum values for the objective function in lieu of performing an excessive number of iterations to obtain truly optimum values can result in a significant reduction in computational time. In all of the aircraft landing-gear reinforcement rings designed by the author using the methods described in this paper, practical minimum values for the objective function have been found adequate and represented great improvement over the best-judgment initial solutions started with.

VI. Example

To illustrate the previously described optimization procedure, the design of a least-weight torque collar for the landing gear of one of a new generation of very large aircraft is described. The geometry of the ring is shown in Fig. 2. Eighteen joints are considered, and it is required that the design be symmetrical about the vertical axis. Although 18 joints have been selected, cross-section properties at joints 1–10 will adequately describe the ring design. Additional joints will be utilized as necessary to indicate the application of loads, define constraints, etc.

The general shape of the cross section at all joints is shown in Fig. 3. The dimensions y and z are variable. If $y > z + 0.75$, then w is the remaining width after a 90° curve, and h is equal to 1.75 in. If $y \leq z + 0.75$, then w is zero and h is the distance from the upper base to the lower boundary of the cross section.

The average thickness of the supporting cylinder is given as 0.30 in. at joints 1–6, 0.3625 in. at joint 7, 0.4625 in. at joint 8, and 0.50 in. at joints 9 and 10.

The applied loads include the following:

1) $p_1 = p_2 = 97{,}000/(6.745 + y_9)$; p_1 and p_2 act vertically downward at joints 8 and 9, respectively.

2) $p_3 = p_4 = -p_1/1.9\ (C_9 + 0.90)$; C_9 is the distance, in inches, from the center to the outer fiber of the section at joint 9. p_3 is applied inward at joint 8 and p_4 is applied outward at joint 9. Both forces are applied in the radial direction.

Table 2 Design variables

Joint	y dimension, in.	z dimension, in.
1	x_1	x_2
2	x_3	x_4
3	x_5	x_6
4	x_7	x_8
5	x_9	x_{10}
6	x_{11}	x_{12}
7	x_{13}	x_{14}
8	x_{15}	x_{16}
9	x_{17}	x_{18}
10	x_{19}	x_{20}

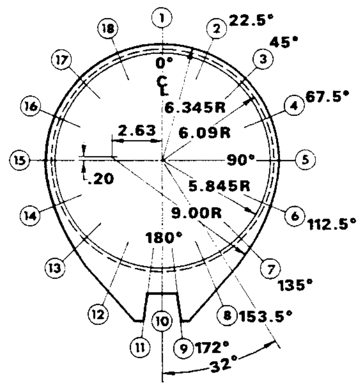

Fig. 2 Example ring (dimensions in inches).

3) p_5 = 160,910 lb acting outward, distributed over 180° of the ring according to a cosine function and centered at joint 15.

The geometrical constraints require the following: 1) the z dimension of each section should not exceed the y dimension; 2) the z dimension of each section should not be less than 0.20 in.; 3) the y dimension at joint 10 should not be less than 0.9903 times the y dimension at joint 9.

The initial values of the variables used are listed in Table 1. These values were considered close to optimum according to the cut-and-try method.

Solution

Design variables

The vector of the design variable is obtained by assigning an x variable to each variable dimension (see Table 2). There are thus twenty variables in this problem, i.e., $m = 20$.

Constraints

Margin-of-safety constraints are written for each of the two ends of each member and 36 safety constraints are thus obtained. The margins of safety are based on 560,000-psi ultimate bending stress, 280,000-psi ultimate tensile stress, 252,000-psi compressive yield stress, and 160,000-psi ultimate shear stress. The following geometrical constraints are also established:

$$G_{37} = x_2 - 0.20 \geq 0,\ G_{38} = x_1 - x_2 \geq 0, \ldots, G_{55} = x_{20} - 0.20 \geq 0,\ G_{56} = x_{19} - x_{20} \geq 0,\ G_{57} = 0.9903x_{17} - x_{19} \geq 0$$

There are thus 57 constraints in this problem, i.e., $n = 57$.

Optimization

The optimization process starts with the given best-judgment initial design:

$$X^{(0)} = \{0.600, 0.400, 0.600, 0.400, 0.600, 0.400, 0.600, 0.400, 0.600, 0.400, 0.600, 0.400, 1.000, 0.600, 2.000, 0.850, 2.250, 1.000, 2.150, 1.000\}$$

The value of the objective function for this design is F = 86.736.

The frame defined by this solution is analyzed and the constraints are evaluated. The axial stresses range from −32,000 to 73,000 psi, bending stresses from −143,000 to

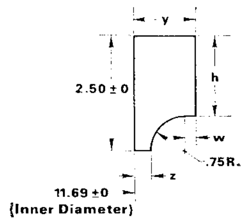

Fig. 3 Geometry of the section at the joints (dimensions in inches).

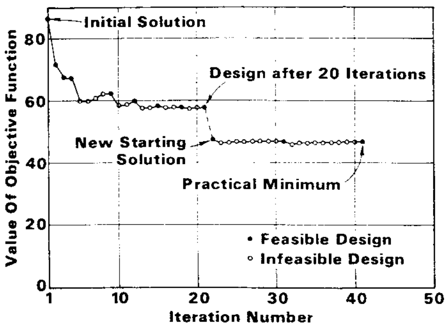

Fig. 4 Variation of the objective function value.

121,000 psi, and shear stresses from −20,000 to 35,000 psi. All the constraints are satisfied by a margin. There are no active or violated constraints. The direction vector is the negative of the unit gradient of the objective function, given by

$$U = \{0.181, 0.043, 0.362, 0.087, 0.362, 0.087, 0.362, 0.087, 0.362, 0.087, 0.365, 0.088, 0.330, 0.100, 0.285, 0.122, 0.207, 0.089, 0.063, 0.027\}$$

The step length λ to the next point $X^{(1)} = X^{(0)} + \lambda \cdot U$ is decided by the first constraint intersected. This intersection is found by modifying trial values of λ, using binary chopping.

Taking a step in the direction of U with a step length $\lambda = 0.547$, the following solution point is reached:

$$X^{(1)} = \{0.501, 0.376, 0.402, 0.353, 0.402, 0.353, 0.402, 0.353, 0.402, 0.353, 0.400, 0.352, 0.820, 0.545, 1.844, 0.783, 0.137, 0.951, 2.116, 0.985\}$$

The value of the objective function for $X^{(1)}$ is $F = 71.77$. The analysis of the frame for the new dimensions shows one active constraint and no violated constraints.

The iteration process is continued until the following solution is obtained in the 20th iteration:

$$X^{(20)} = \{0.386, 0.358, 0.244, 0.244, 0.244, 0.244, 0.244, 0.244, 0.244, 0.244, 0.246, 0.246, 0.615, 0.495, 1.674, 0.710, 2.056, 0.899, 2.035, 0.969\}$$

The value of the objective function at this point is $F = 57.68$.

At this point, the iteration process was terminated due to the limit on the number of iterations. A review of the results had disclosed excessive margin-of-safety values at some of the joints. Engineering judgment suggested that a new initial design, based upon the results of the previous iterations, could be established. A new initial design was defined and the 21st iteration performed based upon this new initial design;

$$X^{(21)} = \{0.200, 0.200, 0.200, 0.200, 0.200, 0.200, 0.250, 0.250, 0.250, 0.250, 0.250, 0.250, 0.600, 0.500, 1.200, 0.600, 1.500, 0.800, 1.400, 0.700\}$$

This point is feasible and has an objective function value of $F = 47.375$.

Nineteen additional iterations are performed beyond $X^{(21)}$ and the following solution is obtained:

$$X^{(40)} = \{0.200, 0.200, 0.200, 0.200, 0.200, 0.200, 0.236, 0.235, 0.236, 0.235, 0.245, 0.235, 0.576, 0.496, 1.179, 0.594, 1.498, 0.797, 1.395, 0.699\}$$

The value of the objective function at the new point is $F = 46.507$.

Based upon engineering judgment, the design $X^{(40)}$ is considered a practical minimum and no further iterations are performed. The variation of the value of the objective function in the iteration process with increasing numbers of iteration is shown in Fig. 4.

The time required on the IBM 360-44 computer is about 1.6 min per iteration.

VII. Summary

A method of designing aircraft landing-gear reinforcement rings for minimum weight is described. The variable dimensions of the cross sections along the ring form the design variables of the problem. The constraints represent the safety conditions and the geometrical restrictions.

In each iteration, the constraints are formed by analyzing the idealized framework for the dimensions obtained at the end of the previous iteration. The change in member forces due to dimension changes within an iteration is neglected.

The optimization technique is a steepest descent method based on vector analysis. Steps are taken toward improved values of the objective function, until a minimum is reached. A practical minimum is obtained by limiting the number of iterations to reduce the computation time.

A computer program has been developed for 1) formulating the objective function, 2) formulating the constraints, and 3) finding the optimum design. The mechanics of the method is illustrated by an example. The design of a least-weight landing-gear torque collar for one of the world's largest airplanes is described. The approach employed is general in nature and may be employed to optimize the design of any reinforcing ring.

References

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[2] Ridha, R. A. and Wright, R. N., "Minimum Cost Design of Frames," *Journal of the Structural Division, Proceedings of the American Society of Civil Engineers*, Vol. 93, No. ST4, Paper 5394, Aug. 1967, pp. 165–183.